Version 1.0

# General Certificate of Education (A-level)
# June 2012

# Mathematics MS03

# (Specification 6360)

# Statistics 3

# Mark Scheme

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**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| $-x$ EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MS03**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | $r$ = **0.665 to 0.666**<br>$r$ = **0.6 to 0.7** | B2<br>(B1) | 2 | AWFW (0.66577)<br>AWFW |
| | **Alternative**<br>Use of a correct formula<br><br>$r$ = **0.665 to 0.666** | (M1)<br><br>(A1) | | $\Sigma x = 20.40$ $\Sigma x^2 = 34.785$ $\Sigma xy =$ **49.1155**<br>$\Sigma y = 28.86$ $\Sigma y^2 = 69.4698$<br>$S_{xx} = 0.105$ $S_{yy} = 0.0615$ $S_{xy} =$ **0.0535** |
| **(b)** | $H_0$: $\rho = 0$<br>$H_1$: $\rho > 0$ | B1 | | Both; do not accept in terms of $r$ but accept in words |
| | SL $\alpha = 0.01$ (1%)<br>CV $r$ = (+)**0.658** | B1 | | AWRT (0.6581)<br>$H_1$: $\rho \neq 0 \Rightarrow$ CV = (±)0.7079 |
| | Calculated $r = 0.666$ > Tabulated $r$ | M1 | | Comparison |
| | **Evidence**, at 1% level, **of a positive correlation** between the neck length and the tail length of mature male giraffes | A1F | 4 | F on $r$ and CV<br>At 1% level, **reject hypothesis of no correlation** |
| | **Total** | | **6** | |
| **2(a)** | $H_0$: $\mu_F = \mu_M$<br>$H_1$: $\mu_F \neq \mu_M$ | B1 | | Both; allow suffices of 1 & 2 or $X$ & $Y$ |
| | SL $\alpha = 0.01$ (1%)<br>CV $z$ = **2.57 to 2.58** | B1 | | AWFW (2.5758) |
| | $z = \dfrac{\bar{f} - \bar{m}}{\sqrt{\dfrac{s_F^2}{n_F} + \dfrac{s_M^2}{n_M}}} = \dfrac{22.0 - 21.6}{\sqrt{\dfrac{1.31^2}{50} + \dfrac{0.702^2}{75}}}$ | M1<br><br>M1 | | Numerator<br><br>Denominator; allow ÷49 and ÷74 but **not** pooling |
| | **1.95 to 2(.0)** | A1 | | AWFW (1.98) |
| | **No evidence**, at 1% level, to suggest that **there is a difference** between mean lengths | A1F | 6 | F on CV and $z$-value<br>allow following pooling |
| **(b)** | Diameter, thickness, girth, width, weight, shape, colour, texture | B1 | 1 | Accept any sensible alternative but **not** 'quality' |
| | **Total** | | **7** | |

**MS03 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)(i)** | P(S $\cap$ U) = 0.15 × 0.10 = **0.015** | B1 | 1 | CAO |
| **(ii)** | P(O $\cap \geq$2) = (0.40 × 0.50) + (0.45 × 0.40) + (0.15 × 0.70) | M1 | | $\geq$1 term correct; may be implied |
| | = 0.20 + 0.18 + 0.105 = **0.485** | A1 | 2 | CAO |
| **(iii)** | P(U) = (0.40 × 0.15) + (0.45 × 0.05) + (0.15 × 0.10) or (i) | M1 | | $\geq$2 terms correct; may be implied |
| | = 0.06 + 0.0225 + 0.015 = **0.097 to 0.098** | A1 | 2 | AWFW (0.0975) |
| **(iv)** | P(D \| U) = $\frac{P(D \cap U)}{P(U)} = \frac{0.40 \times 0.15}{(iii)}$ | M1 | | May be implied |
| | $= \frac{0.06}{0.0975}$ = **0.612 to 0.619** | A1 | 2 | AWFW (0.61538) |
| **(v)** | P(S \| O) = $\frac{0.15 \times (1 - 0.10)}{1 - (iii)} = \frac{0.135}{0.9025}$ | M1<br>M1 | | Numerator<br>Denominator |
| | = **0.149 to 0.15** | A1 | 3 | AWFW (0.14958) |
| **(b)** | P(D $\cap$ T $\cap$ S \| O)<br>$= \frac{0.40 \times 0.85}{1 - (iii)} \times \frac{0.45 \times 0.95}{1 - (iii)} \times (v) \times 3!$ | M1<br>M1<br>M1 | | $\geq$2 terms correct in numerator<br>(1 – (iii)) in denominator<br>3! or 6 or 3 |
| | $= \frac{0.34 \times 0.4275 \times 0.135 \times 6}{0.9025^3}$<br>**or**<br>= 0.3767 × 0.4737 × 0.1496 × 6 | | | |
| | = **0.16** | A1 | 4 | AWRT (0.16016) |
| | **Total** | | **14** | |

**MS03 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4** | $H_0$: $\lambda = 2.6$ (650)<br>$H_1$: $\lambda > 2.6$ (650) | B1 | | Both; accept $\mu$ instead of $\lambda$ |
| | SL $\alpha = 0.05$ (5%) | | | |
| | CV $z =$ **1.64 to 1.65** | B1 | | AWFW (1.6449)<br>Allow 1.96 iff $H_1$: $\lambda \neq 2.6$ |
| | $\hat{\lambda} = \dfrac{688}{250}$ = **2.75(2)** | B1 | | AWRT (2.752)<br>Can be implied by use of 688 |
| | $z = \dfrac{2.752 - 2.6}{\sqrt{\dfrac{2.6}{250}}} = \dfrac{688 - 650}{\sqrt{650}}$ = **1.47 to 1.49** | M1<br>A1 | | Allow use of 2.752 or 688 or 687.5 in denominator<br>AWFW |
| | $p$-value = **0.068 to 0.071** > 0.05 | (M1) | | Use of $P(X \geq 688 \mid \lambda = 650) = 0.072$<br>$\Rightarrow$ M0 A0 (M1) AF1 |
| | **No evidence**, at 5% level, **to support** manager's suspicion | A1F | 6 | F on CV and $z$-value |
| | **Total** | | **6** | |
| **5(a)** | $\hat{p} = \dfrac{68}{125}$ = **0.544** | B1 | | CAO; or equivalent |
| | 98% $\Rightarrow$ $z$ = **2.32 to 2.33** | B1 | | AWFW (2.3263) |
| | Approximate CI for $p$ is<br>$0.544 \pm 2.3263 \times \sqrt{\dfrac{0.544 \times 0.456}{125}}$ | M1<br>A1F | | F on $\hat{p}$ and $z$ |
| | **0.544 ± (0.103 to 0.104)**<br>or<br>**(0.44 to 0.441, 0.647 to 0.65)** | A1 | | CAO/AWFW<br>May be implied by **correct** answer<br>AWFW |
| | **(44%, 65%)** | A1 | 6 | AWRT |
| **(b)** | Require $2 \times 2.3263 \times \sqrt{\dfrac{p(1-p)}{n}} \leq 0.1$ (10%) | M1<br>A1 | | Allow 'no 2' and FT on CI from (a)<br>Allow $p$ = 0.44 to 0.65 |
| | Thus $\sqrt{n}$ = 22.1 to 23.3 | M1 | | Attempted solution for $\sqrt{n}$ or $n$ |
| | ***n* = 490, 495, ..., 545** | A1 | 4 | Must be to 'nearest 5' |
| | **Total** | | **10** | |

**MS03 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)(i)** | $M = U + V$ | | | |
| | $E(M) = 13 + 15 = \mathbf{28}$ | B1 | | CAO |
| | $V(M) = 3^2 + 6^2$ | M1 | | |
| | $+ 2 \times 3 \times 6 \times (-0.6)$ | M1 | | Allow 'no 2' |
| | $= 9 + 36 - 21.6 = \mathbf{23.4}$ | A1 | 4 | CAO |
| **(ii)** | $D = W - 2U$ | | | |
| | $E(D) = 24 - 2 \times 13 = \mathbf{-2}$ | B1 | | CAO |
| | $V(D) = 4^2 + (2^2 \times 3^2)$ | M1 | | |
| | $= 16 + 36 = \mathbf{52}$ | A1 | 3 | CAO |
| **(iii)** | $T = M + W + X$ | | | |
| | $E(T) = 28 + 24 + 9 = \mathbf{61}$ | B1F | | F on $E(M)$ from (a)(i) |
| | $V(T) = 23.4 + 4^2 + 2^2 = \mathbf{43.4}$ | B1F | 2 | F on $V(M)$ from (a)(i) |
| **(b)(i)** | $P(M = 30) = \mathbf{0}$ | B1 | 1 | CAO |
| **(ii)** | $P(W > 2U) = P(D > 0) = P\left(Z > \frac{0-(-2)}{\sqrt{52}}\right)$ | M1 | | Standardising 0 using c's $E(D)$ & c's $V(D)$ from (a)(ii) |
| | $= P(Z > 0.28) = 1 - 0.61026$ | m1 | | Area change |
| | $= \mathbf{0.389 \text{ to } 0.394}$ | A1 | 3 | AWFW (0.39076) |
| **(iii)** | $P(50 < T < 70) =$ | | | |
| | $P\left(Z < \frac{70-61}{\sqrt{43.4}}\right) - P\left(Z < \frac{50-61}{\sqrt{43.4}}\right)$ | M1 | | Standardising 70 & 50 using c's $E(T)$ & c's $V(T)$ from (a)(iii) |
| | $= P(Z < \mathbf{1.37}) - P(Z < \mathbf{-1.67})$ | A1 | | AWRT either |
| | $= 0.91466 - (1 - 0.95254)$ | m1 | | Area change |
| | $= \mathbf{0.865 \text{ to } 0.87}$ | A1 | 4 | AWFW (0.86657) |
| | **Total** | | **17** | |

**MS03 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)(i)** | $X \sim \mathrm{B}(n,\ p)$ | | | |
| | $\mathrm{E}(X) = \sum_{x=0}^{n} x \times \binom{n}{x} p^x (1-p)^{n-x}$ | M1 | | Used; ignore limits until A1 |
| | $= np \times \sum_{x=1}^{n-1} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1}(1-p)^{n-x}$ | M1 | | Factor of $np$ plus $p^x$ to $p^{x-1}$, $n!$ to $(n-1)!$ and $x!$ to $(x-1)!$ Ignore limits until A1 |
| | $= np \times \sum_{x=1}^{n-1} \mathrm{B}(n-1,\ p) = \boldsymbol{np}$ | A1 | 3 | Fully correct and complete derivation AG |
| **(ii)** | $\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$ | | | |
| | $= \mathrm{E}[X(X-1)] + \mathrm{E}(X) - [\mathrm{E}(X)]^2$ | M1 | | Used (Other derivations are possible) |
| | $= n(n-1)p^2 + np - n^2p^2 = \boldsymbol{np(1-p)}$ | A1 | 2 | Fully correct and complete derivation |
| **(b)(i)** | $\binom{n}{m} p^m (1-p)^{n-m} \geq \binom{n}{m-1} p^{m-1}(1-p)^{n-m+1}$ | M1 | | Stated or used |
| | $\frac{n!}{m!(n-m)!} p^{n-m} \geq \frac{n!}{(m-1)!(n-m+1)!}(1-p)^{n-m+1}$ | m1 | | Expanding $\binom{n}{m}$ and $\binom{n}{m-1}$ |
| | $(n-m+1)p \geq m(1-p)$<br>$np - mp + p \geq m - mp$ | A1 | | Simplification |
| | $\boldsymbol{m \leq (n+1)p}$ | A1 | 4 | Fully correct and complete derivation AG |
| **(ii)** | B(10, 0.65) | | | |
| | $(n+1)p = 7.15$ and $(n+1)p - 1 = 6.15$ | M1 | | Can be implied by '7' |
| | $m = \mathbf{7}$ | A1 | | CAO |
| | B(35, 0.5) | | | |
| | $(n+1)p = 18$ and $(n+1)p - 1 = 17$ | (M1) | | Can be implied by '17 or 18' |
| | $m =$ **17 and 18** | A1 | 3 | CAO; accept 'or' |
| **(c)** | $Y \sim \mathrm{B}(4000,\ 0.00095) \approx$ **Po(3.8)** | B1 | | May be implied |
| | $\boldsymbol{k = 3}$ | B1 | | CAO; may be implied |
| | $\mathrm{P}(Y \leq 3) =$ **0.473 to 0.474** | B1 | 3 | AWFW (0.4735) |
| | **SC**: $\boldsymbol{k = 4} \Rightarrow \mathrm{P}(Y \leq 4) =$ **0.667 to 0.668**<br>Award B1 B0 B1 | | | AWFW (0.6678) |
| | **Total** | | **15** | |
| | **TOTAL** | | **75** | |